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Effect of external electric field on dynamics of levitating water droplets

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ABSTRACT

The self-assembled clusters of regularly positioned small droplets were observed for the first time in 2004 by Alexander Fedorets. The recent results on stabilization of droplet clusters enable the laboratory study of specific parameters of chemical and biochemical reactions in the microdroplets. However, the analysis of the cluster behavior under the action of external factors is also continued. Some new experimental findings concerning behavior of a cluster of water droplets levitating over the locally heated water surface are reported in the present paper. It is shown that an external electric field leads not only to an increase in the rate of a condensational growth of droplets but also to a relatively early coalescence of small droplets with water layer. The latter is partially explained by the attraction force arising due to polarization of droplets closely spaced to the water surface. The effect of electric field on size of droplets before their coalescence with water layer is used to estimate an electrical charge of single droplets at the experimental conditions. It is shown that this electric charge does not change the aerodynamic nature of the main forces responsible for both the self-arranging and levitation of small water droplets. The results obtained are expected to be useful for further theoretical modeling of the phenomenon of levitating droplets clusters.

1. Introduction

The fascinating phenomenon of levitating clusters of water microdroplets self-assembled over the locally heated water surface was observed and reported for the first time in 2004 [1]. The levitation of water droplets is supported by an upward vapor flow from the heated surface of a substrate water layer.

It should be recalled that a layer of small droplets over a uniformly heated surface of water like the droplet layer which has been observed by Schaeffer [2] differs significantly from the droplet cluster which is formed only in the case of a local heating of water surface. At the uniform heating, the droplets are positioned randomly (see Fig. 1a), while in the cluster they form a regular structure shown in Fig. 1b.

With fixed maximum water temperature and ambient air properties, the important condition for the formation of a droplet cluster is the size of a hot area on the water surface. This has been confirmed by the laboratory experiments. The general approach to the problem of clustering suggested in recent paper [3] may be of interest in connection with this discussion.

Note that the interest of researchers in the behavior of small water droplets formed from steam above a hot water surface is not limited to the study of a droplet cluster levitating over the water surface. In particular, the authors of paper [4] considered similar water droplets over the adjacent dry surface. This particular problem may be interesting for engineering applications related to cooling of computer processors.

The previous stages of the modeling and experimental research of the droplet cluster have been described in the keynote presentation [5]. Besides the importance of the phenomenon for understanding of fundamental mechanisms of self-assembly, the interest towards the droplet cluster is stimulated by its potential application for chemical and biochemical microreactors [6]. Therefore, all the details of the cluster behavior including small oscillations of single droplets [7] and possible changes in the cluster structure [8] are the subjects of our interest.

Assuming the characteristic time of thermal stabilization of a droplet $\tau_{therm} \approx R^2/a$, where R is the radius of droplet and a is the thermal diffusivity of water. Adopting $R = 40$ μm and $a = 0.15 \cdot 10^{-6}$ m^2/s yields

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Nomenclature

a	thermal diffusivity
E	electric field strength
F	force
e	elementary unit charge
H	height of cluster/droplet levitation
h	distance between droplet and water layer
L	distance between centers of neighboring droplets
l_U	distance between electrodes
m	mass of droplet
P	dipole or multipole moment
q	electric charge
R	radius of droplet
r	radial coordinate
$U^{\pm}$	two configurations of electric field and voltage at the upper electrode
t	current time
u	velocity
W	power
z	axial coordinate

Greek symbols

γ	coefficient in Eq. (7)
ε	dielectric constant
ν	kinematic viscosity
ξ	coefficient in Eq. (8)
ρ	density
τ	relaxation time
φ	electric potential

Subscripts and superscripts

ad	aerodynamic
b	boundary
c	coalescence, critical
el	electrostatic
EK	electrokinetic
dp	dipole
dr	droplet
L	laser
max	maximum
therm	thermal
w	water
z	axial

$\tau_{therm} \approx 0.01$ s. It turns out that the lifetime of the cluster (a few tens of seconds) is long in the terms of the thermal equilibrium of droplets. However, it is restricted because of condensational growing of droplets and the resulting coalescence of large droplets with the substrate layer of water, thus, posing difficulties in the external manipulation of the cluster, crucial for its engineering applications. Therefore, the cluster stabilization at the laboratory conditions was one of the key problems, which has been practically solved [9,10].

It has been well established that aerodynamic forces determine both the formation and evolution of the droplet cluster [5,11,12]. However, an independent estimate of the effect of electric charging the droplets during their condensational growth [13–16] is required to clarify the role of electrostatic Coulomb forces in the cluster. Moreover, an external electric field may affect droplet cluster behavior and its stability. There are two objectives of the present paper: first, we are going to estimate the electrostatic forces between the water droplets in a cluster due to intrinsic electric charge of individual droplets and, second, to perform the experimental observations on the effect of an external electric field on the parameters which characterize the behavior of droplet clusters.

It seems obvious that there are two different physical mechanisms of the influence of an external electric field on droplets of a cluster. First of all, during condensational growth, water droplets are electrified, i.e. acquire their own electric charge. This process was studied, in particular, for the droplets in the atmospheric clouds [17]. Of course, a strong external electric field should lead to the motion of charged water droplets. The registration of such a motion makes it possible to experimentally determine the intrinsic charge of the droplets. The second important effect occurs even for electrically neutral droplets and consists in their polarization in an external electric field. The Coulomb interaction of polarized water droplets with each other is relatively small and cannot affect the structure of the cluster. Of much greater interest is the attraction force of a polarized droplet to a layer of water, which can significantly accelerate the final stage of the coalescence of these droplets with the substrate layer of water. Both of these physical effects of an electric field on cluster droplets are considered in the paper.

Some preliminary results on the relative contribution of the electrostatic and aerodynamic forces on the dynamics of the droplet cluster has been recently reported in Ref. [18]. The current study contains the results of more detailed new experiments, a corrected and completed analysis of the experimental data, and also the experimental data obtained for various shapes of electrodes. The quantitative effect of an external electric field on the conditions of mechanical equilibrium of a levitating droplet is obtained. The intrinsic charges of water droplets just before their coalescence with the water layer are determined and

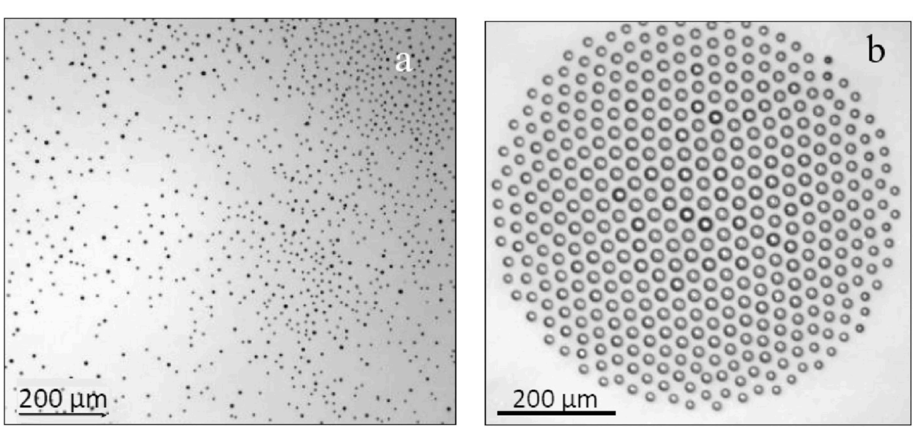

Fig. 1. The droplets over the uniformly heated water surface (a) and the droplet cluster (b).

compared with the typical values for the atmospheric clouds. A contribution of aerodynamic forces to the levitation of droplets is defined more accurately than it was done before.

2. Experimental procedure

The experimental equipment and setup used to produce droplet clusters was described in detail in papers [10,19]. The cluster of water droplets was formed over a thin layer of distilled water on a solid substrate heated locally by laser beam irradiation from below creating a heated spot with the diameter of about 1 mm. The continuous wave semiconductor laser BrixX® 808-800HP (Omicron Laserage, Germany) with the wavelength of 0.808 μm was used. The working power, W_L, was controlled by PM200 device with sensor S401c (Thorlabs, USA). The power of laser radiation affects the local heating of the water layer surface and the evaporation rate. Obviously, the incident radiative flux determines the parameters of the levitating droplet cluster. The value of $W_L = 280$ mW was used in the experiments. The thickness of water layer was equal to 400 ± 2 μm in all experiments. It was controlled using the confocal chromatic sensor IFC2451 made by the company Micro-Epsilon (USA). The cuvette was thermostated at a temperature of 18 ± 0.5°C Video images of the cluster were taken using stereomicroscope Zeiss AXIO Zoom.V16 and high-speed video-camera PCO.EDGE 5.5C (Germany) with spatial resolution of 0.6 μm. To observe the cluster from above, a Zeiss Epi-illuminator Z coaxial vertical reflected-light illuminator was used to provide illumination of the field of vision directly through the microscope objective. To observe the cluster in a lateral projection, the illuminating beam was directed from the side, and the image was constructed using a mirror mounted at an angle close to 45° (see Fig. 2). The distance between images of both the levitating droplet and its reflection in the water layer are used to determine the height of the droplet levitation. The following geometrical parameters of a droplet cluster are also shown in Fig. 2: R is the droplet radius, L is the distance between the centers of neighboring droplets, h is the distance between the droplet and the surface of water layer, $H = h + R$ is the distance between the droplet center and the water layer.

The external electric field was generated by the high-voltage source HVLAB3000 (ET Enterprises, UK), which is designed to vary the applied voltage in the range from 0.2 to 3.0 kV. Both the design and position of electrodes are described below.

2.1. Configuration of electrodes

Three configurations of electrodes used in preliminary experiments are shown in Fig. 3. The variant "a" includes two vertical electrodes made of duralumin plates of thickness 0.2 mm. These electrodes were positioned at almost the same distances from the axis of a droplet cluster. The experiments showed a small horizontal shift of droplets with a decrease of the levitation height in the cluster side which is a little bit closer to one of the electrodes (see Fig. 4). An earlier coalescence of droplets with water layer is observed at this side as compared with the case without external electric field.

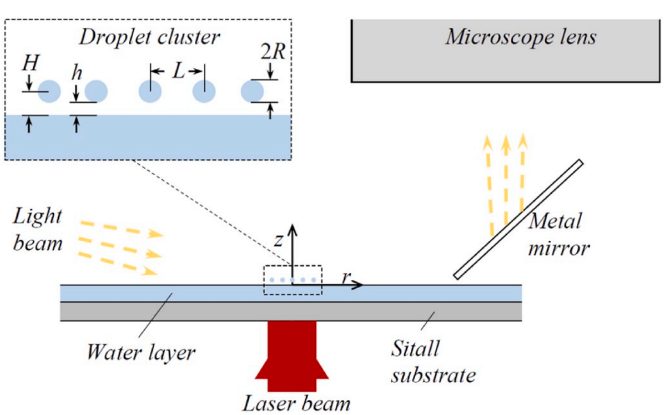

Fig. 2. Schematics of a side view of a laboratory set-up.

In the configuration "b", the lower electrode was a thin metal cylinder (outer diameter – 8.9 mm, the diameter of central orifice 4.5 mm, and thickness – 0.7 mm) placed directly under the sitall substrate, whereas the vertical graphite cylindrical rod of 0.55 mm diameter was used as an upper electrode. The axis of the vertical electrode coincides with the axis of a heating laser beam. The experiments showed that single large droplets move downward and coalesce with water layer, whereas the remaining droplets continue to levitate at a large height above the water layer. On the contrary, the smallest droplets of water move upward to the upper electrode (see Fig. 5). Note that a similar configuration of electrodes was also considered in Ref. [20] where the damping oscillations of water droplets in the external electric field were studied analytically.

In the external electric field, the droplet is subjected to an electrokinetic force [21,22], which is equal to the sum of electrophoretic (EP) and dielectrophoretic (DP) forces:

$$\overrightarrow{F}_{\mathrm{EK}} = q_{\mathrm{dr}}\overrightarrow{E} + \sum_{n=0}^{N}\left(\overrightarrow{P}^{(n)} \cdot \nabla^{n}\right)\overrightarrow{E} \tag{1}$$

where $\overrightarrow{P}^{(n)}$ are nth-order multipole moments and q_{dr} is the droplet charge. The direction of EP force depends on the sign of droplet charge. The DP force is always directed towards a higher electric field strength. From an applied point of view, this force is of interest for working with micro- and nanoscale objects [23]. One can use the data in Fig. 5 to evaluate the excess of the sum of the EK and aerodynamic forces over the droplet weight. The fastest droplet with a radius of 6 μm gains a height of about 0.8 mm in 0.22 s. Obviously, the average droplet acceleration is 0.033 m/s^2, which corresponds to a net force of 0.03 pN. The experiments showed that the described effect does not depend on the electric field direction. This means that the intrinsic electric charge of these small droplets is too small to be taken into account.

The laboratory experiments showed that configurations "a" and "b" are useful to make clear the effects related with polarization of water droplets in the electric field but it is difficult to use these configurations to estimate the own electric charge of the droplets. Therefore, the main study was performed using the configuration "c" [18,24]. In this case, the lower electrode was the same as that in configuration "b", whereas the upper electrode with outer diameter 78 mm and central hole with diameter 6.5 mm was made of fiberglass laminate covered by 18 μm thick copper layer. The distance between the electrodes was equal to $l_{\mathrm{U}} = 8$ mm. For convenience, the variant with a positive electric potential of the upper electrode relative to the grounded lower electrode will be denoted by U^{+}, and the opposite configuration – U^{-}. The experimental results obtained for the main electrode configuration "c" are discussed in section 5.

3. Local electric field and polarization of droplets for the main configuration of electrodes

The intensity of the electric field at the location of droplet cluster for the main configuration "c" of electrodes can be calculated by a numerical solution of the axisymmetric boundary-value problem for the Laplace equation for electric potential $\varphi(r, z)$:

$$\nabla(\varepsilon \ \nabla\varphi) = 0 \tag{2}$$

where $\varepsilon(r, z)$ is the permittivity of substances. The distilled water used in the experiments is a dielectric, in contrast to water with some impurities. The real values of ε for water and substrate substance and obvious boundary conditions for electric potential were used in the calculations reported in Ref. [18]. The finite element method [25,26] with a mesh

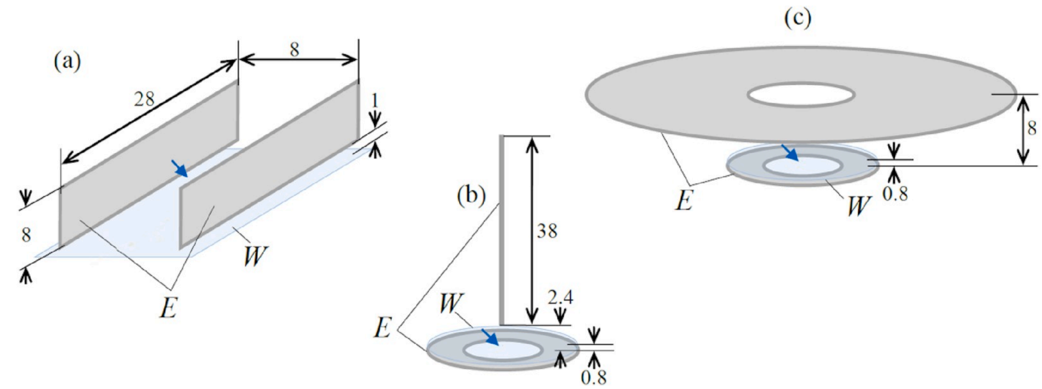

Fig. 3. Schematic configurations of electrodes (E – electrodes, W – upper surface of water layer); blue arrows show the position of droplet cluster. All the dimensions are given in millimeters. (For interpretation of the references to colour in this figure legend, the reader is referred to the Web version of this article.)

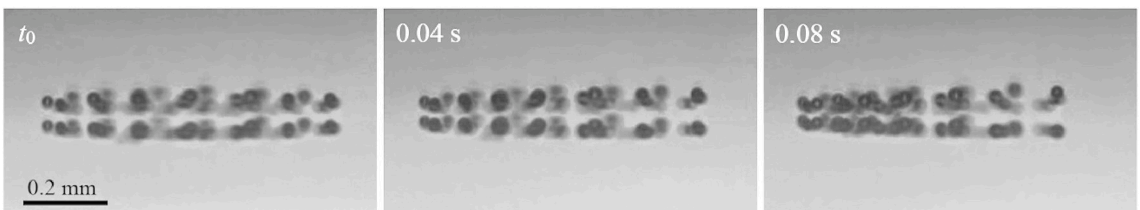

Fig. 4. An asymmetric deformation of droplet cluster in the external electric field of configuration "a" (see Fig. 2) at $U = 1$ kV.

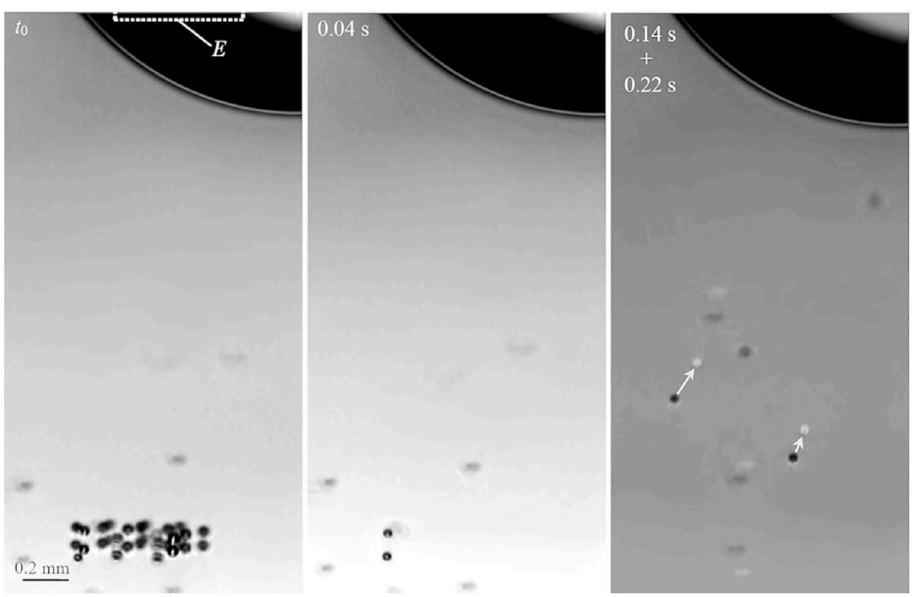

Fig. 5. Effect of external electric field of configuration "b" (see Fig. 3) at $U = 500$ V on water droplets of a cluster. The dark region in the upper right corner of all the images is the edge of a drop of water that grows on the electrode due to steam condensation. The far right image is obtained by superimposing two frames; for clarity, the frame at $t = 0.22$ s is inverted to negative – light droplets correspond to it, the displacement of droplets is shown by white arrows.

containing 3200 triangular elements was employed in these calculations. It was shown that the electric field in the location of cluster is uniform and the absolute value of field strength is equal to $E = |\nabla\varphi| = (U^{\pm}/U_0)\cdot 220$ kV/m where $U_0 = 1$ kV. In the electric field, water droplets are polarized, and the absolute value of the dipole moment of a single spherical droplet with a radius R in a uniform external field is given by Ref. [27]:

$$P = 4\pi\varepsilon_0\frac{\varepsilon - 1}{\varepsilon + 2}R^3E \quad (3)$$

where $\varepsilon_0 = 8.85\cdot 10^{-12}$ F/m is the electric constant. The charges of the upper and lower surfaces of the droplet are the same in their absolute value but opposite in sign, while the surface density of electric charge is proportional to the cosine of the angle measured from the vertical axis. The dipole moment of a droplet is directly proportional to R^3. Therefore, the effects related to the droplet polarization are more pronounced for relatively large droplets.

4. Experimental results for the main configuration of electrodes

The external electric field has a significant effect on the coalescence of droplet cluster with the substrate water layer at is the final stage of the life cycle of the cluster. In the absence of any external electric field, the coalescence has been studied in detail [28,29]. With the condensational growth of water droplets, the distance h between droplets and the water layer decreases. As a result, one of the largest droplets touches the surface of the water layer and generates capillary waves which lead to avalanche-like coalescence of other droplets.

The ordinary fast coalescence of a droplet cluster is illustrated in Fig. 6 where two photographs of the same cluster just before the coalescence and during the coalescence are shown. The temperature of upper surface of water layer under the cluster was equal to 55 °C in this experiment. The video recording with a low frame rate (25 frames per second) led to the appearance of a “phantom” image of the cluster after the fast coalescence which takes about 1 ms only. This effect is the same as that of the so-called spirit photography dating back to the late 19th century.

When an external electric field is applied to the cluster, the coalescence process changes radically for both U^{+} and U^{-} directions of the electric field [18]. This process begins with a significantly smaller droplet size. The sequential coalescence of two water droplets is illustrated in Fig. 7. The coalescence of a single droplet generates capillary waves of small amplitude but without any effect on the neighboring droplets. The time dependences of a distance between droplet and water layer showed the following two stages of the process: the relatively slow decrease in the height of water droplet levitation is changed to the sharp decrease in about 0.05 s before the coalescence [18]. It will be clear from the subsequent analysis that a great acceleration in the vicinity of the water layer surface is explained by the attraction force between the polarized droplet and water layer.

Unlike the coalescence process, the structure of droplet cluster is weakly sensitive to the external electric field. Cluster patterns at $U^{\pm} \leq$ 700 V (to avoid the coalescence) have almost identical distances between the droplets. The detailed measurements for the central part of the cluster at $U^{\pm} = 600$ V reported in Ref. [18] showed only about 1% difference in the values of L as compared with the case of $U = 0$. The experiments showed also the general trend of increasing the rate of growth of water droplets with the electric field intensity, whereas the rate of droplet growth due to condensation is well described by the known d-squared law [30] or its modification called the elliptic law [31]. As to the relatively large growth rate of water droplets in experiments with an external electric, this effect can be explained by the increase in evaporation rate of water layer.

Asakawa reported that a high-voltage electric field directed normally to the water surface increases evaporation rates of distilled water [32], and various applications of this effect are discussed in Ref. [33–36]. While it is difficult to measure the increase in water evaporation in the small hot region just below the cluster, the laboratory measurements of total evaporation from the whole water layer did not indicate any considerable effect of the electric field. The increase in local evaporation rate at the hot water spot could be partially compensated by a continuous coalescence of relatively small droplets with the water layer.

5. Analysis of experimental data

The laboratory experiments showed that the critical size of water droplets, R_c, corresponding to the onset of coalescence decreased in presence of the external electric field. The lower values of R_c are partially explained by an additional force attracting the droplets to the water layer. In contrast to Ref. [18], the cluster was recorded in lateral projection for a longer time, which made it possible to trace all the significant stages of the effect of the electric field U^{+} on the behavior of cluster droplets. New measurements showed that an external electric field first increases the levitation height, H, of small droplets (due to their intrinsic negative electric charge), and then, as the droplet radius, R, increases due to steam condensation, the electric field contributes to a significant decrease in the levitation height and earlier coalescence of the droplet with substrate water layer. For the convenience of working with the experimental data, the dependence of the droplet weight, $mg = (4/3)\pi R^3 \rho_w g$, on the levitation height is presented in Fig. 8. One can distinguish three different periods of the downward motion of a water droplet:

- The initial period (the right-hand side of the graph), when the droplet of water is very small and the electric field does not act on it because of the small intrinsic charge of the droplet and because of its extremely insignificant dipole moment;
- The longest (in height) second period during which (due to the large distance of the droplet from the water surface) the dipole interaction of the droplet with the water layer is insignificant and the influence of an external field on the gradually increasing intrinsic charge of the droplet predominates (this is a range of about $28 < H < 39$ μm);
- The third period ending the trajectory of the droplet, when a relatively large droplet with a significant dipole moment is very close to the surface of water layer ($H/R < 1.6$) and the attraction of a polarized droplet to the water layer leads to its relatively early coalescence.

The measurements made during the second period of the droplet movement can be used to determine the intrinsic charge of water

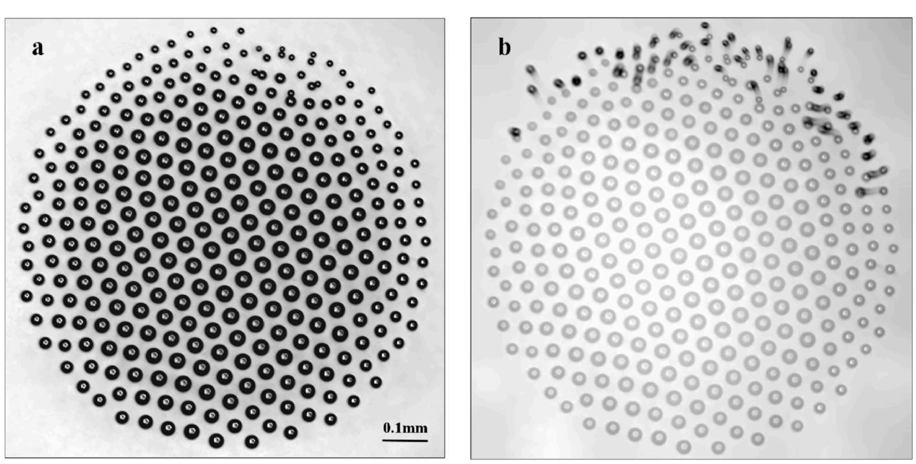

Fig. 6. The images of a droplet cluster without an external electric field: (a) just before the coalescence and (b) during the coalescence (with “phantom” images of disappeared large droplets).

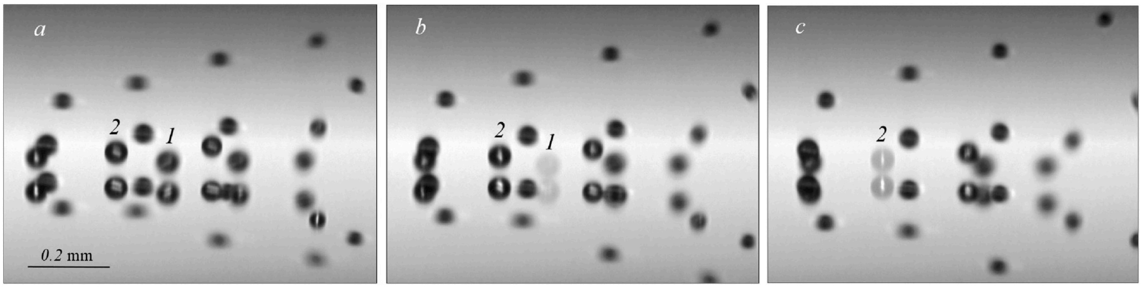

Fig. 7. Sequential coalescence of droplets 1 and 2 with water layer in external electric field at $U^{+} = 500$ V: a – both droplets levitate at $t = t_0$; b – the phantom of droplet 1 is observed at $t = t_0 + 0.04$ s, c – the phantom of droplet 2 is observed at $t = t_0 + 0.08$ s.

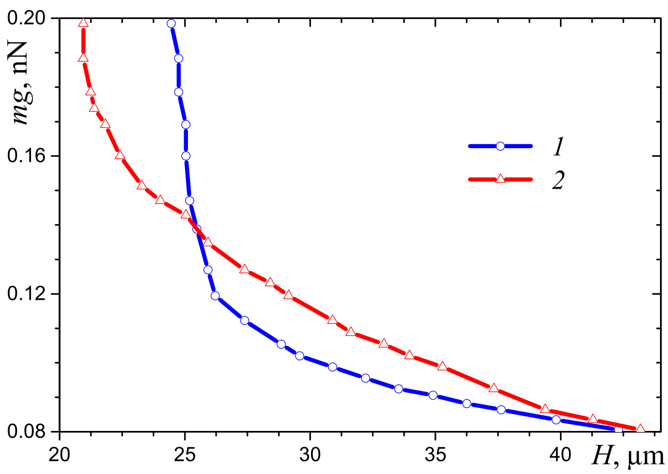

Fig. 8. Experimental dependences of the weight of a levitating droplet on the levitation height: 1 – without external electric field ($U = 0$), 2 – in the external electric field at $U^{+} = 600$ V.

droplets. These results are more complete and accurate in comparison with those reported in Ref. [18]. The positive difference of $\Delta(mg) = mg|_{U^{+}>0} - mg|_{U=0}$ (see Fig. 8) can be used to determine the negative electric charge of the droplet. The absolute value of this charge is calculated as follows:

$$q_{dr} = \Delta(mg) / E \quad (4)$$

In the case of $U^{+} = 600$ V, the electric field strength is equal to $E =$ 132 kV/m, and simple calculations show the monotonic increase in the value of q_{dr} with the droplet radius. For convenience, the ratio of q_{dr}/e, where $e = 1.6{\cdot}10^{-19}$ C is the elementary unit charge, is presented in Fig. 9. The experimental points correspond to the decrease of levitation height from $H = 39.4$ μm to $H = 28.4$ μm. The droplet growing continues also at smaller values of H, but it is impossible to determine the intrinsic electric charge of the larger droplets because of a significant contribution of the attractive force between the polarized droplet and water layer. So, the maximum value of $q_{dr} = 740$ e for water droplets with radius of $R = 14.4$ μm is obtained from the reported new experimental data. The radius of droplets in clusters generated in the absence of an external electric field is usually several times larger. It means that one can expect the value of $q_{dr} > 10^3 e$ for these droplets. On the other side, one cannot exclude a contribution of the partially ionized steam in the laboratory experiments.

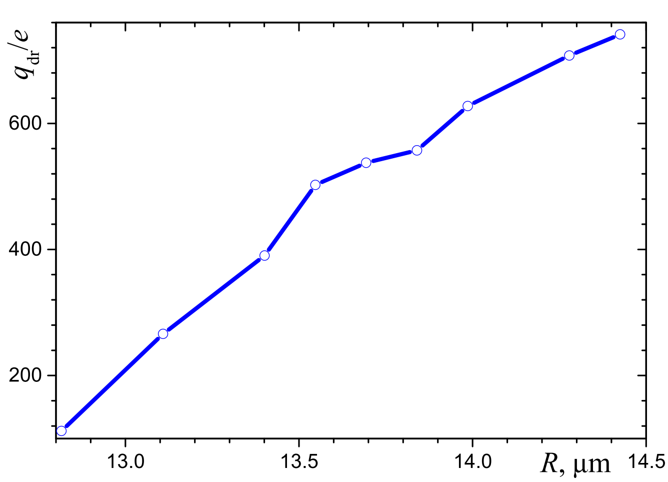

Fig. 9. Experimentally determined variation of the intrinsic droplet charge with the radius of water droplet.

The obtained values of droplet charge in clusters are in good agreement with an estimate of q_{dr}~$10^2 - 10^3 e$ obtained in papers [13, 14] using quite different methods. It should be recalled that charged water droplets are the ordinary objects in the lower atmosphere [37–40]. So, the laboratory studies of the effect of electrical charging on behavior of small droplets and their clusters are also interesting for geophysical applications. Note that our estimation of electric charge of water droplets is also in good agreement with the data for atmospheric clouds. For weakly charged droplets, most researchers working in the field of atmospheric electricity agree that the droplet charge is usually directly proportional to the square of the droplet radius, and the coefficient is in the range from 10^{-6} to $5{\cdot}10^{-6}$ C/m^2 [17]. For charge of $q_{dr} = 740$ e, this coefficient can be estimated as about $5.6{\cdot}10^{-7}$ C/m^2. A possible source of this "insufficient charge" of cluster droplets compared to those in clouds may be a strong flow of partially ionized steam from the substrate layer of water. The latter can be explained by the higher electrical conductivity of hot steam compared with the conductivity of relatively cold moist air in atmospheric clouds [41].

The attraction force between the water layer and a polarized droplet can be estimated assuming that the opposite charges of the dipole are located at the upper and lower points of the droplet surface. At small values of the levitation height, h, in comparison with the droplet radius, the effect of the upper charge can be ignored. Then the following equations for the absolute value of dipole electric charges and the attracting force between the dipole and water layer are valid:

$$q_{dp} = \frac{P}{2R} = 2\pi\varepsilon_0 \frac{\varepsilon - 1}{\varepsilon + 2} R^2 E \quad (5a)$$

$$F_{dp}^{max} = \frac{1}{4\pi\varepsilon_0} \frac{\varepsilon - 1}{\varepsilon + 1} \frac{q_{dp}^2}{4h^2} = \pi\varepsilon_0 \frac{\varepsilon - 1}{\varepsilon + 1} \left(\frac{\varepsilon - 1}{\varepsilon + 2}\right)^2 E^2 \frac{R^4}{4h^2} \quad (5b)$$

Note that Eq. (5b) is a revised version of a similar equation in Ref. [18]. The main error of this equation is related to a complex distribution of the electric charge on the droplet surface. In addition, both opposite charges of the polarized droplet affect the attraction force between the droplet and water layer. Fortunately, there is no need in the accurate calculations of the attractive force in the most interesting case of $h < R$, when Eq. (5) gives a correct estimate.

It is also important that the polarization charge, q_{dp}, in the laboratory experiment is much greater that the own charge of the droplets. The dependences of the force on the levitation height, $F_{dp}^{max}(h)$, were calculated (Fig. 9). These dependencies show a strong increase of the attracting force with decreasing the distance between typical polarized droplets and a layer of water. It is important that F_{dp}^{max} is comparable with the gravitational force at small values of h (compare Fig. 10 and Fig. 8). For a stable cluster, the greatest aerodynamic drag force is equal to the sum of the gravitational force and the discussed electric attracting force. However, a contribution of the electric force is considerable at small distances from the surface of water layer, just before the droplet coalescence.

The regular structure of a self-arranged droplet cluster observed in the experiments is relatively stable. As we have already noted, at $U^{\pm} =$ 600 V, the values of L are almost the same as those without the electric field. This indicates that additional forces between the neighboring droplets due to their polarization are very weak and they do not affect the cluster structure. However, this argument is insufficient to explain the small effect of electric field. It is interesting to recall that the distances between droplets in the ordinary cluster (without an external electric field) are very sensitive to the steam generation rate controlled by the laser heating power. The strong influence of the electric field on the droplets growth rate can explain a very small effect of electric field on distances between droplets.

The Reynolds number $\mathrm{Re} = 2u_0R/\nu$, where $u_0 \approx 0.1$ m/s is the velocity of steam at the water layer surface and ν is the kinematic viscosity of steam, is one of the main parameters, which determine the interaction regime between the steam flow and water droplets. The value of ν decreases slightly with temperature from about $7.2{\cdot}10^{-4}$ m^2/s at 50 °C to $6.1{\cdot}10^{-4}$ m^2/s at 70 °C. For droplets of radius about 40 μm we obtain $\mathrm{Re} \approx 0.1$. This small Reynolds number is typical of the Stokes flow regime characterized by a linear momentum equation. However, the resulting simplification is insufficient to simplify radically the general problem.

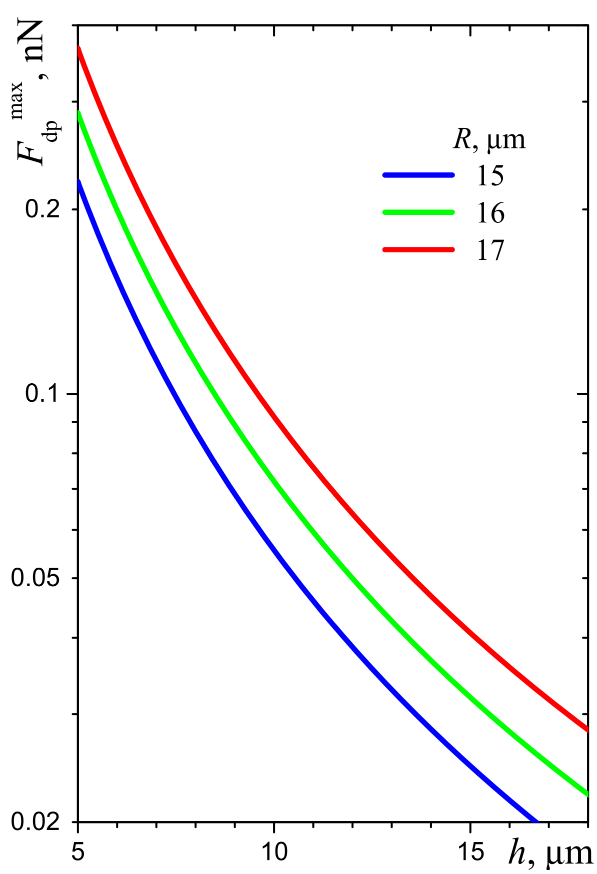

Fig. 10. The calculated electric force attracting polarized droplets to water layer.

Unfortunately, the computational modeling of the gas flow and forces between the droplets is very complicated due to the complex spatial arrangement of water droplets in the cluster, the competitive effects of steam condensation and water evaporation, and diffusion in the steam-air mixture. Our physical estimates are based on some relatively simple solutions because detailed computational modeling is impossible at present.

For two identical spherical droplets or particles held fixed side by side against a uniform gas flow perpendicular to the line connecting their centers, the interaction was analyzed computationally in papers [42,43]. For the range of $10 \le \mathrm{Re} \le 150$, calculations showed that the two spheres repel each other when the spacing is of the order of the diameter, and the repulsion is stronger at smaller spacing between the particles. On the other hand, the two particles weakly attract each other at the intermediate separation distances. The numerical results of [43] for Re = 10 and 50 were slightly corrected in Ref. [44] and additional results for smaller distances between the particles showed a significant increase in the repelling force. The calculations of [43] in the Stokes regime showed that the particles weakly repel each other at all separations.

The interaction of two particles in the uniform flow is different from the same in the droplet cluster. The most important difference is caused by the small height of levitation of the cluster in comparison with clusters' diameter. Our analysis is limited to a comparison of the repelling Coulomb force between the neighboring charged water droplets and approximate values of the aerodynamic attraction force between the same droplets. For two interacting droplets, the electrostatic force is given by:

$$F_{el} = \frac{1}{4\pi\varepsilon_0}\frac{q_{dr}^2}{L^2} \tag{6}$$

Equation (6) yields the value of $F_{el} \approx 0.19$ pN at $q_{dr} = 740\ e$ and $L =$ 80 μm.

The forces acting on the cluster droplets from the steam flow can be significant, especially for large droplets, when the strong flow leads to a considerable repulsion of neighboring droplets from each other. As shown in Ref. [8], this can even lead to a change in the cluster configuration. In this paper, we consider only small droplets of a specific cluster in the presence of a strong external electric field. Therefore, in the estimates given below, only the attractive force between adjacent drops appears, explained by Bernoulli's law for flow in a narrowing channel. The aerodynamic force due to decrease in static pressure of a moving gas between the droplets can be estimated as:

$$F_{ad} = \frac{\rho\left(u_1^2 - u_0^2\right)}{2}\xi{\cdot}4\pi R^2 \tag{7}$$

where u_1 is the average velocity in the gap between the droplets, and $\xi <$ 1 is a dimensionless coefficient corresponding to a conventional part of the droplet surface subjected to the attraction force. For the hexagonal structure of droplet cluster, the value of $u_1 > u_0$ can be calculated as follows:

$$u_1 = \ u_0\Big/\gamma \ \gamma = 1 - \frac{2\pi}{\sqrt{3}}\frac{R^2}{L^2} \tag{8}$$

where the coefficient γ takes into account the reduced cross-sectional area of steam flow due to the presence of droplets in the hexagonal droplet cluster. In the case of $R << L$, we obtain:

$$F_{ad} = \xi\ \ \rho u_0^2\frac{8\pi^2}{3}\frac{R^4}{L^2} \tag{9}$$

Note that, the ratio of F_{el}/F_{ad} does not depend on the distance between water droplets because both F_{el} and F_{ad} are inversely proportional to L^2. At the same time, the absolute value of the minimum aerodynamic force is comparable or greater than the electrostatic force. Assuming $\rho = 0.65$ kg/m^3, $u_0 = 0.08$ m/s, $R = 20$ μm, and $L = 80$ μm in (9), we obtain $F_{ad} = 0.27$ pN at $\xi = 0.1$. Note that Eq. (9) differs from the similar formula of paper [18], which contains an error in the coefficient.

Our estimations confirm that aerodynamic forces are dominant over the electrostatic forces, and therefore, the aerodynamic forces are responsible for the interaction between the droplets in the levitating cluster. A much more sophisticated analysis would be required to understand the observed spatial scale of the regular pattern in the self-assembled cluster. The expected complexity of this analysis is clear from the recently observed transition from hexagonal to chain structure of self-arranged droplet clusters [8].

At first glance, it might seem that the period of the regular structure of the droplet cluster is related to the equilibrium between the attractive and repulsive forces of different physical nature acting between the droplets. In fact, this equilibrium is almost entirely determined by aerodynamic forces. The last statement becomes clear from the following qualitative reasoning. The flow rate of steam is rather high and the flow cannot completely bend around a wide droplet cluster located at a small height above the water surface. A significant portion of the steam flow passes between the droplets. In the case of a regular hexagonal structure of a cluster, the steam flow does not allow the droplets to be too close to each other. On the other hand, the reduced static pressure in the minimum cross section of the flow between the droplets (because of a relatively high velocity) creates an attractive force that keeps the droplets at a distance comparable with their own size. Thus, the balance of the horizontal components of the forces acting on the drop is formed by the conditions of the flow of steam in the space between the droplets.

6. Conclusions

We performed laboratory experiments with the droplet clusters levitating over the locally heated water surface in an external electric field. Several configurations of the electric field were checked in the experiments. The configuration with the electrodes parallel to the surface of water layer appeared to be the most convenient one to obtain both the qualitative and quantitative results on behavior of water droplets. In contrast to our previous paper [18], the motion of a droplet was registered along the main part of its trajectory. As a result, three different periods of the downward motion of a water droplet were distinguished for the first time. The second of these periods enabled us to estimate the variation of the intrinsic electric charge with the radius of droplets. This is much more detailed information than an average intrinsic electric charge estimated in Ref. [18]. It is clear now that the negative electric charge of droplets in the central part of the cluster grows with increasing droplet size, and under experimental conditions this charge reaches about 740 elementary electric charges. The analysis of the final period of droplet motion showed that polarization of water droplets causes an additional attraction force between the droplets and water layer. This force is comparable to the gravitational force at very small height of a droplet levitation and contribute to the early coalescence of small droplets with the water layer. The corrected analytical estimations based on the experimental data confirmed that electrostatic force between the droplets is negligible in comparison with the dominating aerodynamic force and the aerodynamic forces control the cluster pattern.

Declaration of competing interest

None declared.

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